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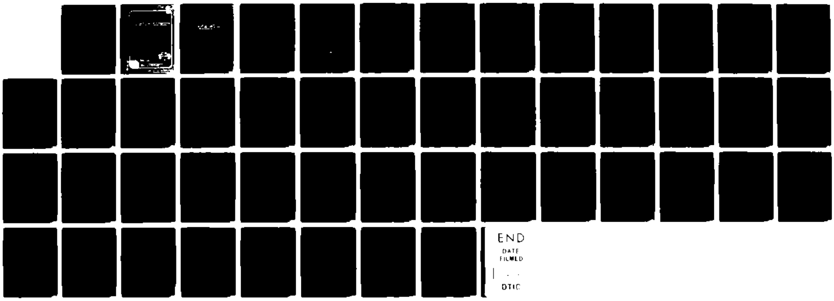

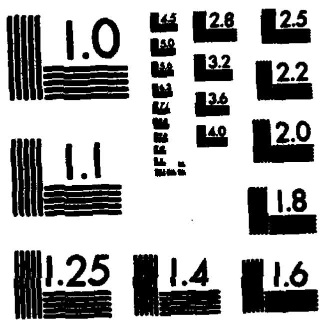

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Although the combat capabilities of alternative aircraft carrier designs have been much analyzed, force structure comparisons for peacetime uses, such as the projection of presence, are less often quantified. This paper uses a Blotto game framework to evaluate equal cost forces of alternative carrier designs in the presence role.

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A GAME-THEORETIC MEASURE OF PRESENCE FOR ASSESSING AIRCRAFT CARRIER OPTIONS

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October 1982

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PREFACE

Although the combat capabilities of alternative aircraft carrier designs have been much analyzed, force structure comparisons for peacetime uses, such as the projection of presence, are less often quantified. This paper uses a Blotto game framework to evaluate equal cost forces of alternative carrier designs in the presence role.

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A. INTRODUCTION

Aircraft carriers currently constitute the chief offensive component of the U.S. Navy's general purpose fleet. As new classes of carriers have been designed and built, the displacement of these ships has steadily increased, so that today's Nimitz-class carrier has a full-load displacement of over 90,000 tons. Carrier costs have also increased, and today the procurement cost of a Nimitz-class ship is well over $3 billion,[1] not counting the aircraft that make up its air wing. One obvious consequence of high carrier cost is that large carriers cannot be procured in large numbers. The consequent size of the U.S. Navy has decreased over time; today the Navy consists of 12 carrier battle groups. Even with the large military expenditures being contemplated by the current administration, the number of carrier battle groups is not expected to grow beyond 15 before the end of the century.

In the meantime, the number of areas of the world where the U.S. would like to exert influence through part-time or full-time naval presence appears to be growing. The Chief of Naval Operations (CNO) has cited the following regions as among those in which the U.S. has interests: the Mediterranean, the Indian Ocean, the South China Sea, the Japan-Korea area, and the Caribbean (Reference [2]). Since it requires, as a rule-of-thumb, about three carriers in the fleet to support one carrier deployed to a forward station, it is not difficult to understand why

[1]All costs in this paper are given in Fiscal Year (FY) 1982 constant dollars.

the CNO maintains that the Navy is currently "stretched thin around the globe."

One suggestion that is often put forth when this apparent mismatch of offensive platforms to commitments is discussed is to build less costly carriers, of which more could be procured. For a fixed budget, therefore, one could acquire a force structure with greater dispersal capability. However, less costly carriers are by necessity smaller carriers, with fewer aircraft on each.

One problem with this suggestion is that it is generally held that large carriers are more efficient for operating aircraft than small carriers. Therefore, on an equal cost basis, a fleet of smaller carriers supports fewer total sea-based aircraft, and fewer aircraft on a per ship basis, than a fleet of large carriers. The loss in overall capability is tolerable only if the advantages of a greater number of carriers somehow outweigh the capability reduction. This paper addresses a major peacetime role of carriers--maintaining presence in forward areas--and investigates the properties of a game theoretic measure of fleet capability in that role. The paper then compares a number of naval force structures based on ship designs that have been discussed in the past. The next section provides an overview of our approach, while Section C addresses how the measure is computed. Section D provides some results and conclusions are presented in Section E.

B. AN OVERVIEW OF THE MEASURE

Many models, simulations, and analytical techniques exist that assess the capability of naval forces in conflict, but no similar literature is evident for measuring the capability of naval forces to disperse aircraft into a number of different geographical areas.[1] The procedure which we employ makes use of results from the Theory of Games applied to a gedanken contest between two carrier forces--a baseline force (denoted C, which determines the location of the origin for this measure) and the force under consideration (denoted R). The contest is this: there are Ω independent oceans, or regions, in which it is considered important to establish presence. We assume that the two naval forces are in the hands of two strategists who, acting simultaneously, allocate their forces among the Ω areas (see Figure 1). The outcome of the contest is determined by awarding points according to the following rules:

(1) If, in a given area, there are one or more carriers from force R and no carriers from force C, force R is credited with 1 point.

(2) If, in a given area, there are one or more carriers from force C and none from force R, force R loses one point.

(3) If, in a given area, carriers from both forces are present, then R receives 1 point if the total of force R aircraft is greater than the total of force C aircraft, loses one point if the total of force C aircraft is greater, and neither receives nor loses a point if the totals of aircraft are equal.

(4) If neither force is represented in an area, no points are awarded or deducted.

The sum of the points over all of the Ω areas (which may be negative) is the outcome of the contest.[2] It is assumed that the interests of the two strategists are directly antithetical--the one employing force R wants the sum of points as large as possible and the one employing force C wants the sum to be as small as possible. The game is, of course, zero-sum.

We will not use the outcome of a single contest as our measure, but rather we propose to use the value of the game

[1]We will refer to the capability to introduce forces into different areas as "presence" to distinguish the measure presented here from typical effectiveness measures based on warfare simulations.

[2]This is obviously a version of the well-known "Colonel Blotto" game.

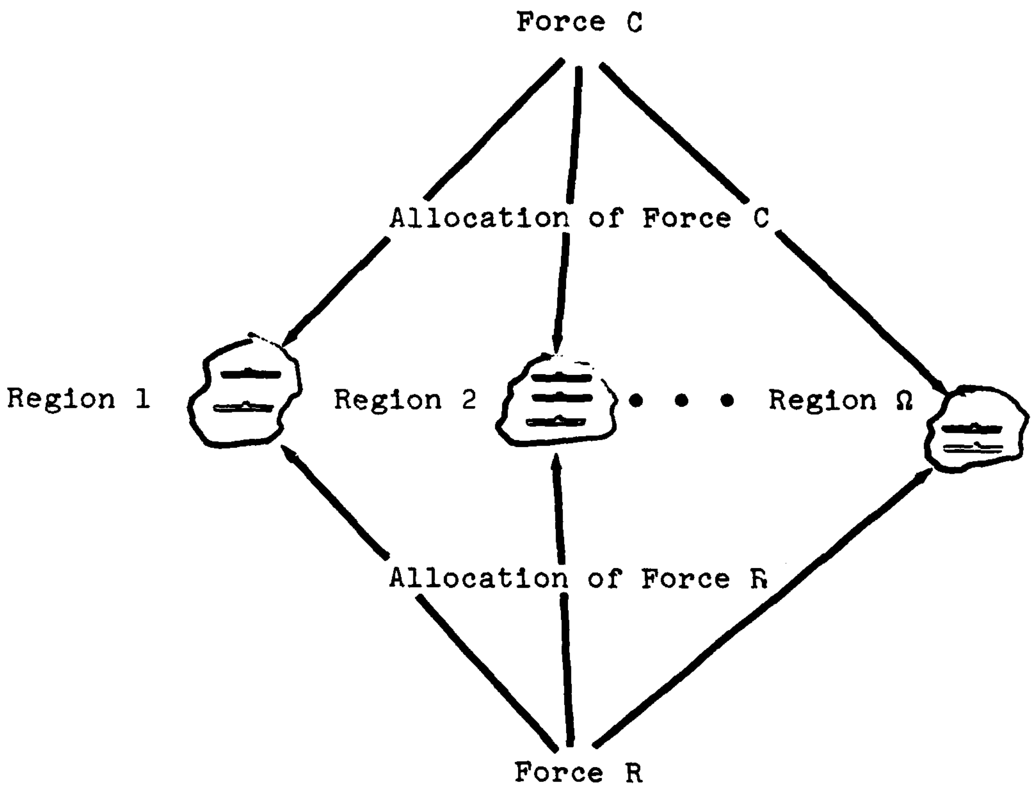

Figure 1. FORCE ALLOCATIONS

(in the traditional game theoretic sense) as defined by John von Neumann and Oskar Morgenstern (Reference [6]). The value is that number of points, V, that the force R strategist can guarantee himself by "behaving optimally" irrespective of what his opponent does.[1] (It is a theorem of Game Theory that, by behaving "optimally," the force C strategist can hold his opponent to no more than V points.)

As an example, suppose the baseline force C consists of one large carrier with 90 aircraft, the force R to be evaluated consists of three small carriers with 25 aircraft each and that there are 3 areas of interest. Then we can form a payoff matrix as shown in Figure 2. The allocations are given in the form (a b c) where a is the number of carriers allocated to area 1, b is the number of carriers allocated to area 2 and c is the number of carriers allocated to area 3. The pure strategies of

Allocations of Force R	Allocations of Force C: (1 0 0)	(0 1 0)	(0 0 1)
(3 0 0)	-1	0	0
(0 3 0)	0	-1	0
(0 0 3)	0	0	-1
(2 1 0)	0	0	1
(2 0 1)	0	1	0
(0 2 1)	1	0	0
(1 2 0)	0	0	1
(1 0 2)	0	1	0
(0 1 2)	1	0	0
(1 1 1)	1	1	1

Value = 1

Figure 2. EXAMPLE PRESENCE GAME PAYOFF MATRIX

[1]The notion of "behaving optimally" involves the concepts of mixed strategy which we discuss briefly in the following sections. More complete discussions can be found in any standard game theory text.

this game are thus specific allocations of ships to areas. Although not even all three small carriers can compete with one large one in terms of numbers of aircraft, force R obviously has a presence measure, relative to force C, of 1, obtainable by allocating one small ship to each area; that is, because the value of the game is positive, force R has superior presence qualities.

In the game depicted in Figure 2, force R has an obvious choice of strategy. In general this will not be the case, and the game value will be predicated on the assumption that each side can randomize over its choice of strategies. In this case, the value of the game is a true expected value where each side has optimized its "mixed strategy," i.e., its probability distribution over allocation choices.

Before discussing computational aspects of this measure, it is appropriate to note some of its features.

First, the measure takes into account the major parameters of presence operations: the number of areas of concern, the number of carriers available to meet those concerns, and the capability of those carriers. While we measure that capability solely in terms of aircraft numbers for this paper, there is no fundamental reason why the capability characteristics of those aircraft could not be included into the entries of the payoff matrix.

Indeed, there are excellent reasons for doing so. As carrier size decreases, so does the ability to operate large, high performance aircraft. Carriers above approximately 60,000 tons can operate all modern sea-based aircraft including F-14 interceptors and A-6 medium attack aircraft, the most capable of offensive carrier aircraft; a carrier with displacement between about 40,000 tons and 60,000 tons can operate some conventional (catapult launched, and arrested landing) aircraft such as the light attack/fighter F/A-18 but not heavier, more capable, aircraft. Below about 40,000 tons, carriers are restricted to helicopters and vertical/short take-off and landing (VSTOL) aircraft, such as the AV-8B Harrier, the only operational VSTOL military aircraft. Current VSTOL designs are range- and payload-limited compared to conventional aircraft of similar size. Thus, by examining only aircraft numbers, we are ignoring the penalty in aircraft capability imposed by smaller carriers. For presence operations, however, this penalty may not be as serious as it would be in conflict situations. Determining aggregate measures of aircraft capability is a major effort in itself and beyond the scope of this paper. The reader should nevertheless bear in mind that aircraft capability is an important factor in the large carrier/small carrier controversy.

Second, the procedure used in this paper, which assesses one U.S. force structure against another U.S. force structure, is a somewhat unorthodox approach. Traditionally, analyses of U.S. capability pit a U.S. force against a projected threat, say a Soviet force, within certain scenario assumptions. Idealized engagements are proposed and simulated and the outcomes generally determine some measure of effectiveness. Presence, however, does not involve actual combat (although it may evolve into conflict); it is more a matter of perception than quantifiable capability. Moreover, while an opponent may have naval forces present in an area, only the U.S. possesses general purpose carriers to any degree, so that opposing forces will necessarily be qualitatively much different. How to determine the outcome of a "presence confrontation" between a U.S. carrier battle group and, say, a Soviet task force made up of cruisers and destroyers (with perhaps the potential for support from land-based bombers) is not at all clear. But, since the U.S. believes in establishing presence with carrier battle groups, and the function of those battle groups is to support sea-based air power, it does not appear at all inappropriate to compare the ability of two force structures to support aircraft in a number of areas by evaluating them against each other. This yields the game that underlies this measure.

Third, in this paper we compare only pure force structures; that is, force structures with only a single carrier type. In theory, there is no reason why mixed force structures cannot be handled. Indeed there is good reason to do so since many of the carriers in the present U.S. Navy can be expected to remain in service for considerable lengths of time and thus would remain a factor if new ship types are procured. The reason for examining pure forces is two-fold. First, identification of optimal pure (i.e. uniform) forces suggests what the Navy should ultimately look like, 30, 40, or 50 years from now, with mixed forces providing only a (relatively) short-term transitional consideration. Second, pure forces greatly simplify the computational aspects of evaluating this measure, a consideration in any computational technique.

Finally, some may object to the notion of optimal mixed strategy, which is intimately connected with the concept of game-theoretic value, since a mixed strategy implies that force allocations should be determined randomly, in general, and without knowledge of opposing allocations at the time the allocation is made. This is not the only way to view the game value, however. An alternative characterization of the game value is as the limit of the average return for a sequence of iterated games with "each player choosing in turn the best pure strategy against the accumulated mixed strategy of his opponent up to then" (Reference [4]). That is, at a given move in the sequence of games, a player examines the previous moves of his opponent, treats them as a mixed strategy, and chooses his own best pure strategy in response. Reference [4] established that the average payoff of this game per iteration when repeated to infinity converges to the game value. This sequence of optimized response and counter-response is perhaps more in keeping with the idea of two strategists attempting to outweigh each other in various parts of the world.

C. COMPUTING THE MEASURE

Although it is a straightforward application of linear programming to find the value of the game outlined in the previous section, the large number of different allocations that results from arranging even a moderate number of ships into a few different areas quickly makes the programming problem very large. This is a common difficulty with Blotto games and has been dealt with variously by "*convexifying*" payoff functions (Reference [1]), or by specifing particular payoff functional forms (Reference [5]), which then lead to tractible methods of solutions, or through some other simplifying assumption. Our approach exploits some of the symmetries of the Blotto game of the previous section to generate a smaller, related game. Knowing the value of the smaller game yields the value of the original game in a straightforward manner. In this section, we outline this process.[1]

The following notation and terminology will be used. For an $m \times n$ matrix $A = (a_{ij})$, the i^{th} row will be denoted by A_{i*}, $i = 1, \ldots, m$; the j^{th} column denoted by A_{*j}, $j = 1, \ldots, n$. Given vectors $x = (x_1,\ldots,x_m)$ and $y = (y_1,\ldots,y_n)$, the scalar product is used to write

$$x \cdot A_{*j} = \sum_{k=1}^{m} x_k a_{kj} \ ,$$

$$y \cdot A_{i*} = \sum_{\ell=1}^{n} y_\ell a_{i\ell} \ .$$

[1] This approach, which employs obvious symmetry and dominance relationships among the rows of columns of the game matrix, is not intrinsically new. The basic elements of this approach have long been part of the literature of game theory (see, for example, Karlin's 1959 game theory book [3]). This section illustrates the remarkable reduction in game size made possible by systematic repeated application of these two characteristics.

Force R will be called the row player, and force C will be called the column player.

To show that two games have the same, or related, value, we use the following basic fact from game theory:

> A game with the m×n payoff matrix $A = (a_{ij})$ has value V if and only if there are mixed strategies[1] $x = (x_1,\ldots,x_m)$ and $y = (y_1,\ldots,y_n)$ such that
>
> $$(1) \quad x \cdot A_{*j} \geq V \ , \quad j = 1, \ldots, n \ ;$$
> $$(2) \quad y \cdot A_{i*} \leq V \ , \quad i = 1, \ldots, m \ .$$

A useful interpretation of these conditions is:

(1) for each column, the row player has an expected payoff of at least V;[2]

(2) for each row, the column player has an expected loss no greater than V.[2]

To illustrate the reduction technique, a specific example will be worked out. The above necessary and sufficient condition will be used to show that each successive reduction of the original game transforms the game value in an elementary manner.

In this example, there are 3 oceans. Force C has 6 large carriers with 90 aircraft on each. Force R has 11 small carriers with 40 aircraft on each. The payoff matrix and the strategies of each force are shown in Figure 3.

---

[1]A mixed strategy is a vector $x = (x_1,\ldots,x_r)$ satisfying

$$\sum_{j=1}^{r} x_j = 1, \ x_j \geq 0 \ , \quad j = 1, \ldots, r \ .$$

The x_j are called weights.

[2]We speak of payoffs to the row player from the column player. A negative payoff, therefore, is a payoff to the column player from the row player.

OCEAN #1 #2 #1 #2 #3	6 0 0 0 6 0 0 0 6	5 0 1 1 5 0 0 1 5	5 1 0 0 5 1 1 0 5	4 0 2 2 4 0 0 2 4	4 2 0 0 4 2 2 0 4	4 1 1 1 4 1 1 1 4	3 0 3 3 3 0 0 3 3	3 1 2 2 3 1 1 2 3	3 2 1 1 3 2 2 1 3	2 2 2
11 0 0 0 11 0 0 0 11	-1 0 0 0 -1 0 0 0 -1	-2 -1 0 0 -2 -1 -1 0 -2	-2 0 -1 -1 -2 0 0 -1 -2	0 -1 0 0 0 -1 -1 0 0	0 0 -1 -1 0 0 0 -1 0	-1 -1 -1 -1 -1 -1 -1 -1 -1	0 -1 0 0 0 -1 -1 0 0	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1
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6 1 4 4 6 1 1 4 6	1 1 1 1 1 1 1 1 1	-1 1 1 1 -1 1 1 1 -1	1 1 -1 -1 1 1 1 -1 1	-1 -1 1 1 -1 -1 -1 1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-3 1 -1 -1 -3 1 1 -1 -3	-1 -1 -1
6 3 2 2 6 3 3 2 6	1 1 1 1 1 1 1 1 1	1 -1 1 1 1 -1 -1 1 1	-1 1 1 1 -1 1 1 1 -1	-1 -1 1 1 -1 -1 -1 1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 -1 1 1 -1 -1 -1 1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-3 -1 1 1 -3 -1 -1 1 -3	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1
6 2 3 3 6 2 2 3 6	1 1 1 1 1 1 1 1 1	-1 1 1 1 -1 1 1 1 -1	1 1 -1 -1 1 1 1 -1 1	-1 -1 1 1 -1 -1 -1 1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-3 1 -1 -1 -3 1 1 -1 -3	-1 -1 -1
5 5 1 1 5 5 5 1 5	1 1 1 1 1 1 1 1 1	1 -1 1 1 1 -1 -1 1 1	-1 1 1 1 -1 1 1 1 -1	1 -1 1 1 1 -1 -1 1 1	-1 1 1 1 -1 1 1 1 -1	-1 -1 1 1 -1 -1 -1 1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 1 1 -1 -1 -1 1 -1	-1 -1 1 1 -1 -1 -1 1 -1	1 1 1
5 4 2 2 5 4 4 2 5	1 1 1 1 1 1 1 1 1	1 -1 1 1 1 -1 -1 1 1	-1 1 1 1 -1 1 1 1 -1	-1 -1 1 1 -1 -1 -1 1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 -1 1 1 -1 -1 -1 1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-3 -1 1 1 -3 -1 -1 1 -3	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1
5 2 4 4 5 2 2 4 5	1 1 1 1 1 1 1 1 1	-1 1 1 1 -1 1 1 1 -1	1 1 -1 -1 1 1 1 -1 1	-1 -1 1 1 -1 -1 -1 1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-3 1 -1 -1 -3 1 1 -1 -3	-1 -1 -1
5 3 3 3 5 3 3 3 5	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1	-1 -1 1 1 -1 -1 -1 1 -1	-1 1 -1 -1 -1 1 1 -1 -1	1 1 1 1 1 1 1 1 1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 1 1 -1 -1 -1 1 -1	-1 1 -1 -1 -1 1 1 -1 -1	-1 -1 -1
4 4 3 3 4 4 4 3 4	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	1 1 1 1 1 1 1 1 1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1 -1 -1	-3 -3 -3

Figure 3. PAYOFF MATRIX FOR EXAMPLE WITH 6 LARGE CARRIERS, 11 SMALL CARRIERS

In figure 3, the strategies have been divided into 26 row groups and 10 column groups. The payoff matrix has been partitioned into blocks. For each block, the respective row player and column player strategies are generated by the shift permutation (e.g., 920,092,209). Note that while most blocks are size 3x3, the last column has blocks of size 3x1. For a given 3x3 block, all row sums and column sums are the same. Note also that each 3x1 block contains identical entries.[1]

The first reduction is accomplished by forming a 26x10 matrix whose $(i,j)^{th}$ entry is any column sum of the corresponding $(i,j)^{th}$ block. The resulting matrix is shown in Figure 4. Beside each row and column is a representative strategy chosen from the corresponding group of strategies. The first strategy in each block of 3 strategies is the one selected.

To show that this matrix represents an "equivalent" game, the above necessary and sufficient condition is used as follows. Suppose the mixed strategies

$$x' = (x_1, \ldots, x_{26})$$

and

$$y' = (y_1, \ldots, y_{10})$$

solve the reduced game (Figure 4) with value V. Then the mixed strategies

$$x = \left(\frac{x_1}{3}, \frac{x_1}{3}, \frac{x_1}{3}, \ldots, \frac{x_{26}}{3}, \frac{x_{26}}{3}, \frac{x_{26}}{3}\right) ,$$

---

[1]When a player's number of ships is divisible by the number of oceans, precisely one strategy forms a group by itself. If the row player had 12 ships, the last row strategy would be 444. The bottom row of the payoff matrix would consist of blocks of size 3x1, and one block of size 1x1.

OCEAN		#1	6	5	5	4	4	4	3	3	3	2
		#2	0	1	0	2	0	1	3	2	1	2
#1	#2	#3	0	0	1	0	2	1	0	1	2	2
11	0	0	-1	-3	-3	-1	-1	-3	-1	-3	-3	-3
10	1	0	1	-2	-2	0	0	-3	0	-3	-3	-3
10	0	1	1	-2	-2	0	0	-3	0	-3	-3	-3
9	2	0	1	-2	-2	-1	-1	-4	0	-3	-3	-3
9	0	2	1	-2	-2	-1	-1	-4	0	-3	-3	-3
9	1	1	3	-1	-1	0	0	-4	1	-3	-3	-3
8	3	0	1	0	0	-2	-2	-1	0	-1	-1	-3
8	0	3	1	0	0	-2	-2	-1	0	-1	-1	-3
8	2	1	3	-1	-1	-1	-1	-5	1	-3	-3	-3
8	1	2	3	-1	-1	-1	-1	-5	1	-3	-3	-3
7	4	0	1	0	0	-2	-2	-1	0	-1	-1	-3
7	0	4	1	0	0	-2	-2	-1	0	-1	-1	-3
7	3	1	3	1	1	-1	-1	-1	1	-1	-1	-3
7	1	3	3	1	1	-1	-1	-1	1	-1	-1	-3
7	2	2	3	-1	-1	-1	-1	-5	1	-3	-3	-3
6	5	0	1	0	0	0	0	-1	-4	-1	-1	3
6	0	5	1	0	0	0	0	-1	-4	-1	-1	3
6	4	1	3	1	1	-1	-1	-1	-3	-3	-3	-3
6	1	4	3	1	1	-1	-1	-1	-3	-3	-3	-3
6	3	2	3	1	1	-1	-1	-1	-3	-3	-3	-3
6	2	3	3	1	1	-1	-1	-1	-3	-3	-3	-3
5	5	1	3	1	1	1	1	-1	-3	-1	-1	3
5	4	2	3	1	1	-1	-1	-1	-3	-3	-3	-3
5	2	4	3	1	1	-1	-1	-1	-3	-3	-3	-3
5	3	3	3	3	3	-1	-1	3	-3	-1	-1	-3
4	4	3	3	3	3	-3	-3	3	-3	-3	-3	-9

Figure 4. FIRST REDUCTION OF PAYOFF MATRIX

and

$$y = \left(\frac{y_1}{3},\frac{y_1}{3},\frac{y_1}{3},\ldots,\frac{y_9}{3},\frac{y_9}{3},\frac{y_9}{3},y_{10}\right)$$

will solve the original game, with value V/3. By denoting the matrix of Figure 4 as A', this last assertion is verified by observing that

$$x'\cdot A'_{*j'} \geq V \text{ and } y'\cdot A'_{i'*} \leq V \text{ for all } i',j'$$

if and only if

$$x\cdot A_{*j} \geq V/3 \text{ and } y\cdot A_{i*} \leq V/3 \text{ for all } i,\ j\ .$$

Thus, the value of this first reduced game immediately yields the value of the original game.

The next reduction deletes the extra copies of any repeated rows and columns. Notice that in Figure 4, when two strategies differ by a permutation (e.g., 920 and 902, or 321 and 312), their associated rows or columns are identical. The first copy of each repeated row or column is kept. The additional copies are deleted. The matrix resulting from these deletions is shown in Figure 5. The associated strategies are also shown. These strategies can be listed in a canonical fashion; namely, each is a non-increasing distribution of ships into 3 oceans. Table 1 lists the canonical distributions for the row and column players. Table 2 indicates, for various values of N and Ω, how many canonical distributions of N ships into Ω oceans exist.

The matrices in Figure 4 and Figure 5 produce the same game value. Moreover, mixed strategies which yield value V in Figure 5 can be used to do the same in Figure 4. The key point is that the sum of the weights assigned to identical rows or columns in Figure 4 must equal the weight on the corresponding row or column in Figure 5. For example if

OCEAN		#1	6	5	4	4	3	3	2
		#2	0	1	2	1	3	2	2
#1	#2	#3	0	0	0	1	0	1	2
11	0	0	-1	-3	-1	-3	-1	-3	-3
10	1	0	1	-2	0	-3	0	-3	-3
9	2	0	1	-2	-1	-4	0	-3	-3
9	1	1	3	-1	0	-4	1	-3	-3
8	3	0	1	0	-2	-1	0	-1	-3
8	2	1	3	-1	-1	-5	1	-3	-3
7	4	0	1	0	-2	-1	0	-1	-3
7	3	1	3	1	-1	-1	1	-1	-3
7	2	2	3	-1	-1	-5	1	-3	-3
6	5	0	1	0	0	-1	-4	-1	3
6	4	1	3	1	-1	-1	-3	-3	-3
6	3	2	3	1	-1	-1	-3	-3	-3
5	5	1	3	1	1	-1	-3	-1	3
5	4	2	3	1	-1	-1	-3	-3	-3
5	3	3	3	3	-1	3	-3	-1	-3
4	4	3	3	3	-3	3	-3	-3	-9

Figure 5. SECOND REDUCTION OF PAYOFF MATRIX: ONLY CANONICAL DISTRIBUTIONS APPEAR

Table 1. CANONICAL DISTRIBUTIONS FOR ROW AND COLUMN PLAYERS

COLUMN PLAYER: 6 CARRIERS, 3 OCEANS

6	0	0
5	1	0
4	2	0
4	1	1
3	3	0
3	2	1
2	2	2

ROW PLAYER: 11 CARRIERS, 3 OCEANS

11	0	0
10	1	0
9	2	0
9	1	1
8	3	0
8	2	1
7	4	0
7	3	1
7	2	2
6	5	0
6	4	1
6	3	2
5	5	1
5	4	2
5	3	3
4	4	3

EACH DISTRIBUTION IS A NON-INCREASING PARTITION OF 6 OR 11 CARRIERS INTO 3 OCEANS.

Table 2. VALUES FOR $T(N,\Omega)$ = NUMBER OF CANONICAL DISTRIBUTIONS OF N CARRIERS INTO Ω OCEANS: $N \leq 25$, $\Omega \leq 10$

N=	Ω = 1	2	3	4	5	6	7	8	9	10
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	2	2	2	2	2	2	2	2	2
3	1	2	3	3	3	3	3	3	3	3
4	1	3	4	5	5	5	5	5	5	5
5	1	3	5	6	7	7	7	7	7	7
6	1	4	7	9	10	11	11	11	11	11
7	1	4	8	11	13	14	15	15	15	15
8	1	5	10	15	18	20	21	22	22	22
9	1	5	12	18	23	26	28	29	30	30
10	1	6	14	23	30	35	38	40	41	42
11	1	6	16	27	37	44	49	52	54	55
12	1	7	19	34	47	58	65	70	73	75
13	1	7	21	39	57	71	32	89	94	97
14	1	8	24	47	70	90	105	116	123	128
15	1	8	27	54	84	110	131	146	157	164
16	1	9	30	64	101	136	164	186	201	212
17	1	9	33	72	119	163	201	230	252	267
18	1	10	37	84	141	199	248	288	318	340
19	1	10	40	94	164	235	300	352	393	423
20	1	11	44	108	192	282	364	434	488	530
21	1	11	48	120	221	331	436	525	598	653
22	1	12	52	136	255	391	522	638	732	807
23	1	12	56	150	291	454	618	764	887	984
24	1	13	61	169	333	532	733	919	1076	1204
25	1	13	65	185	377	612	860	1090	1291	1455

To generate $T(N,\Omega)$, we have:

For $\Omega = 1$, $T(N,1) = 1$ for all N.

For $\Omega \geq 2$,

$$T(N,\Omega) = \sum_{j=0}^{[N/\Omega]} T(N-j\Omega,\Omega-1) ,$$

where $[N/\Omega]$ = greatest integer $\leq N/\Omega$.

$$\hat{y} = (\hat{y}_1, \ldots, \hat{y}_7)$$

is a mixed strategy for the column player in Figure 5, yielding value V, then

$$y' = \left(\hat{y}_1, \frac{\hat{y}_2}{2}, \frac{\hat{y}_2}{2}, \frac{\hat{y}_3}{4}, \frac{3\hat{y}_3}{4}, \hat{y}_4, \hat{y}_5, \frac{\hat{y}_6}{11}, \frac{10\hat{y}_6}{11}, \hat{y}_7\right)$$

will yield value V in Figure 4.

The matrix in Figure 5 is almost 20 times smaller than the original payoff matrix. Yet, for an example with 5 oceans, 11 large carriers, and 25 small carriers, such a reduced matrix would have dimensions 377 x 37. This is still fairly large. Significant savings can be achieved by employing one last series of reductions, for which the following terms are defined.

In an m×n matrix $A = (a_{ij})$, row i dominates row ℓ if $a_{ik} \geq a_{\ell k}$ for k = 1, ..., n. One also writes that row ℓ is dominated by row i. Similarly, columns of A may dominate or be dominated by one another.

This last series of reductions involves successively deleting dominated rows and dominating columns. To show why this results in an equivalent game, the objectives of the row player are examined. The row player chooses a mixed strategy which gives the largest minimum expected return, all columns considered. In Figure 5, row 2 dominates row 1. Thus, the row player could always improve his expected return from each column by transferring any positive weight of a mixed strategy from row 1 to row 2. Hence, the row player can effectively ignore row 1, and any other dominated rows. To verify that the deletion of dominated rows results in an equivalent game, observe that

$$\hat{y} \cdot A_{1*} \leq \hat{y} \cdot A_{2*} .$$

In other words, the column player's expected loss from a dominated row is less than or equal to the expected loss from the associated dominant row.

By deleting dominated rows, one obtains a game with the same game value and with no row domination. The same reasoning as above allows the deletion of dominating columns. The resulting matrix might now have dominated rows, so this cycle is repeated. This reduction stops when the matrix has no row or column domination. In certain cases, this last matrix will be of dimensions 1x1, and will itself yield the value of the game.

For the current example, the successive row-reduced and column-reduced matrices are shown in Figures 6-10. The last matrix, in Figure 10, is the final reduced payoff matrix. It is smaller than the original payoff matrix by a factor of 364. In large examples, this factor has been as high as 3000.

To compute the game value, a linear program, based on the matrix in Figure 10, is solved. A mixed strategy solution is

$$\tilde{x} = (2/3,\ 1/3),\ \tilde{y} = (0,1,0) \quad ,$$

and the value is -1. The value of the original game, therefore, is -1/3.

We close this section by describing how this reduction technique is actually implemented. It is a simple matter to generate the reduced matrix of Figure 5 directly. First, the canonical distributions for the two players are generated (see Table 1). Let $\alpha^i = (\alpha^i_1,\ldots,\alpha^i_\Omega)$ be the i^{th} canonical row player distribution, and $\beta^j = (\beta^j_1,\ldots,\beta^j_\Omega)$ be the j^{th} canonical column player distribution. Let $\langle \alpha^i_k, \beta^j_\ell \rangle$ denote the outcome (+1,0,or -1) of an engagement between α^i_k small carriers and β^j_ℓ large carriers. The matrix $A = (a_{ij})$ of Figure 5 is then given by

$$a_{ij} = \sum_{k=1}^{\Omega}\sum_{\ell=1}^{\Omega} <\alpha_k^i,\beta_\ell^j> .$$

In turn, dominated rows and dominating columns are deleted. When no more deletions are possible, a linear program is solved. The resulting value, V, gives the value of the original game, V/Ω.

Ocean #1 #2 #3	6 0 0	5 1 0	4 2 0	4 1 1	3 3 0	3 2 1	2 2 2
10 1 0	1	-2	0	-3	0	-3	-3
9 1 1	3	-1	0	-4	1	-3	-3
7 3 1	3	1	-1	-1	1	-1	-3
5 5 1	3	1	1	-1	-3	-1	3
5 3 3	3	3	-1	3	-3	-1	-3

Figure 6. ROW-REDUCTION OF MATRIX IN FIGURE 5: DOMINATED ROWS HAVE BEEN DELETED

Ocean #1 #2 #3	4 1 1	3 3 0	3 2 1	2 2 2
10 1 0	-3	0	-3	-3
9 1 1	-4	1	-3	-3
7 3 1	-1	1	-1	-3
5 5 1	-1	-3	-1	3
5 3 3	3	-3	-1	-3

Figure 7. COLUMN-REDUCTION OF MATRIX IN FIGURE 6: DOMINATING COLUMNS HAVE BEEN DELETED

Ocean #1 #2 #1 #2 #3	4 1 1	3 3 0	3 2 1	2 2 2
7 3 1	-1	1	-1	-3
5 5 1	-1	-3	-1	3
5 3 3	3	-3	-1	-3

Figure 8. ROW-REDUCTION OF MATRIX IN FIGURE 7

Ocean #1 #2 #1 #2 #3	3 3 0	3 2 1	2 2 2
7 3 1	1	-1	-3
5 5 1	-3	-1	3
5 3 3	-3	-1	-3

Figure 9. COLUMN-REDUCTION OF MATRIX IN FIGURE 8

Ocean #1 #2 #1 #2 #3	3 3 0	3 2 1	2 2 2
7 3 1	1	-1	-3
5 5 1	-3	-1	3

Figure 10. ROW-REDUCTION OF MATRIX IN FIGURE 9:
THIS IS THE FINAL REDUCED PAYOFF MATRIX

D. RESULTS

The methods described in the previous section were applied to a variety of ship, air wing, and area combinations. In all cases, the baseline force is four carriers with ninety aircraft apiece. If we assume a three-to-one ratio of total battle groups to forward-deployed battle groups, the baseline is representative of a twelve battle group Navy, more-or-less what exists today. Figures 11 through 14 summarize the results for from 4 to 15 deployed task forces allocated over from 3 to 7 areas.[1] (Note that the results for 5 and 6 areas are identical and combined into a single Figure.) In each Figure an oddly shaped form is displayed. This form is the zero set (ZS): that is, it encompasses those combinations of task force numbers and air wing sizes that yield zero game values against the baseline force. (Actually, only integer combinations make sense, but we have connected those combinations with straight lines to make the ZSs more visible.) Directly above the ZSs lie those combinations that yield a positive game value; that is, that are superior to the baseline force according to our measure. Directly below the ZSs are those combinations that yield a negative game value; that is, that are inferior to the baseline force.

Note that the ZSs suggest (and it is simple to prove) that:

(1) For a given number of areas and a given number of deployed task forces, the game value increases monotonically[2] with air wing size.

(2) For a given number of areas and a given air wing size the game value increases monotonically with the number of deployed task forces.

It is <u>not</u> the case, however, as can be seen from a careful comparison between Figures 11, 12, and 13 (examine the Table in

[1] The results depicted in Figures 11-14 are tabulated in Appendix B.
[2] Monotonicity need not be strict.

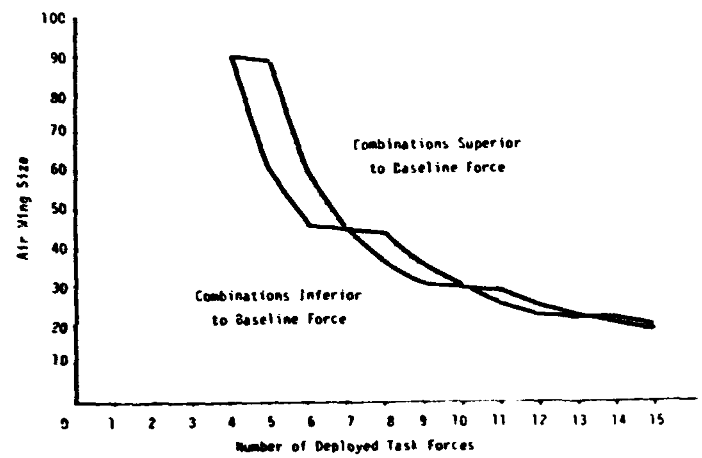

Figure 11. COMPARISONS TO BASELINE FORCE, 3 AREAS

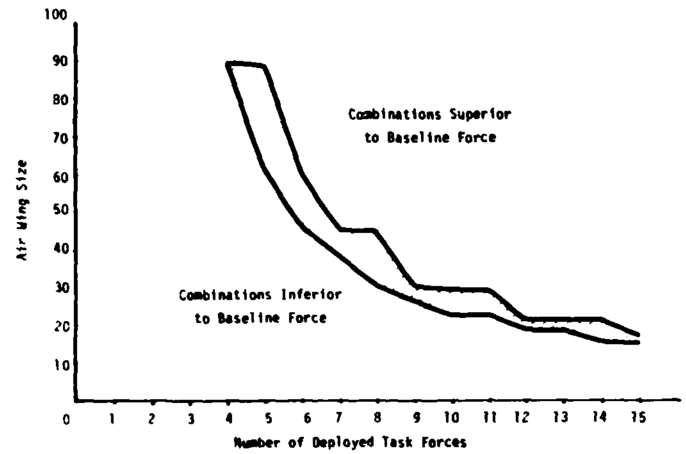

Figure 12. COMPARISONS TO BASELINE FORCE, 4 AREAS

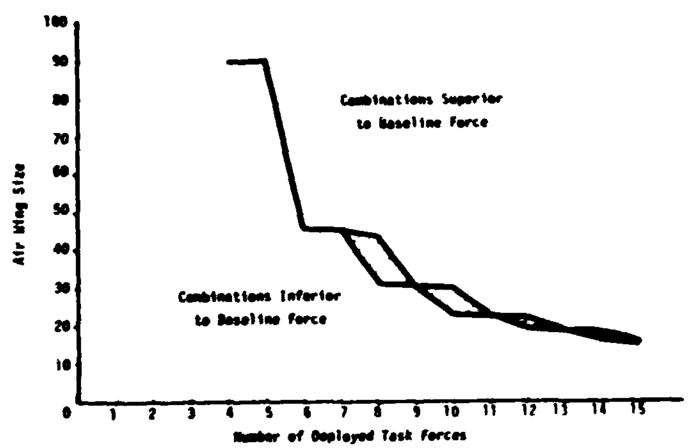

Figure 13. COMPARISONS TO BASELINE FORCE, 5,6 AREAS

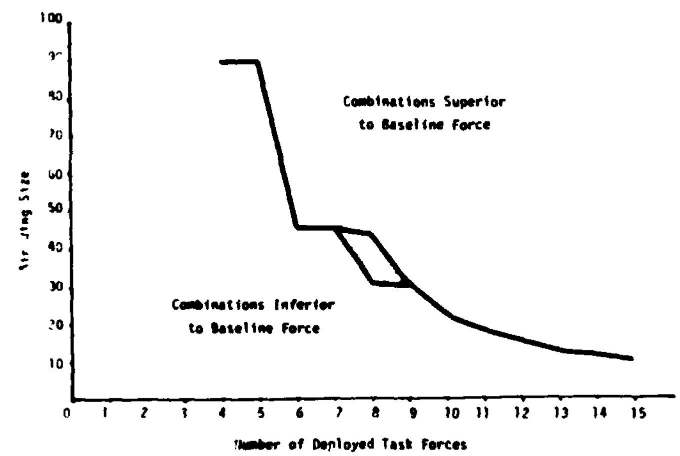

Figure 14. COMPARISONS TO BASELINE FORCE, 7 AREAS

Appendix B for the combinations of 8 task forces, 45 aircraft in 3, 4 and 5 areas), that for a given number of task forces and a given air wing size, the game value behaves monotonically with the number of areas. While the irregularities of the ZSs make generalizations difficult, there appears to be a tendency for the right-hand portions of the ZSs to swing down as the number of areas increases. This results in, for example, a combination that produced a negative game value for 3 areas yielding a positive game value in 7 areas (e.g., 14 task forces, 19 aircraft). This is consistent with the intuitive notion that smaller task forces in greater numbers generally become more desirable as the number of areas increases.

While none of the above is particularly insightful--most of the observations discussed above could be arrived at without resorting to the computations of Section C--the utility of these results is that they allow quantifiable preference distinctions to be made among force structures. This procedure is illustrated in the remainder of this section.

Table 3 lists a variety of hypothetical force structures based on different air-capable ships. The force structures are oriented toward presence missions since they include fewer defensive escorts per air-capable ship than are dictated by current Navy plans.

The CVN is, of course, the nuclear-powered Nimitz-class carrier currently in production. The remaining options have been proposed in Congressional testimony or in other open sources over the last 10 years or so. The CVV is a medium-size, 62,000 ton, conventionally propelled carrier that has been proposed in recent years as a more affordable carrier option. The Light Carrier (LC) is an even smaller carrier that would operate perhaps 35 CTOL or VSTOL aircraft. The Through-Deck Cruiser (TDC) is an air-capable ship with additional armament--guns and missiles--in contrast to the other

Table 3. SUMMARY OF EQUAL LIFE CYCLE COST FORCE STRUCTURES

Carrier Type	Carrier Air Wing (No. Aircraft)	Carrier Displacement (Tons)	Number and Type of Escorts per Task Force	30-Year Life Cycle Cost Including Underway Replenishment (Billions of FY82 Dollars)	Deployable Task Forces Under Equal Cost Constraint
Nimitz Class Carrier (CVN)	90	91,000	2 CGN-42	43.1	4
CVV Medium Carrier	60	62,000	2 DDG-51	29.9	5.8
Light Carrier (LC)	35	39,000	2 DDG-51	19.0	9.1
Through-Deck Cruiser (TDC)	27 (VSTOL)	45,000	1 DDG-51	16.6	10.4
VSTOL Support Ship (VSS)	25 (VSTOL)	26,000-29,000	2 DDG-51	16.4	10.5
Sea Control Ship (SCS)	17 (VSTOL)	14,000	2 DDG-51	11.7	14.7

options which are exclusively ships for operating aircraft with minimal additonal (mostly defensive) armament. The VSTOL Support Ship (VSS) and the Sea Control Ship (SCS) are ships on the small end of the carrier spectrum. Each task force comprises one air-capable ship and two general purpose escorts, except for the TDC which, because it carries armament of its own, is accompanied by only one escort. The Nimitz is escorted by two hypothetical nuclear-powered Aegis cruisers to complement the nuclear propulsion of that carrier, while the other task forces would utilize the proposed conventionally propelled DDG-51 destroyers--a future design that incorporates Aegis technology. The life cycle cost figures include underway replenishment support for the task forces. These costs are based on a number of assumptions and come from several different sources and are therefore only approximate. Nonetheless, we feel they are useful for comparing these options. Further details concerning these alternatives and the costing assumptions are discussed in Appendix A.

Figures 15-18 repeat Figures 11-14 but display the equal cost force structures of Table 3. For 3 areas, none of the alternatives is superior to the CVN force (which is more or less today's Navy). As the number of areas increases, certain alternatives begin to appear more desirable. For 4 areas, the Light Carrier force yields a combination of task force numbers and aircraft that is superior to the baseline force. For 5 and 6 areas, the CVV force also becomes superior while for 7 areas, all of the alternative force structures except the CVN force is superior to the baseline.

Thus, once a determination is made concerning the number of areas of the world in which presence is desired, this procedure allows cost effective force structures for the presence mission to be identified. Those results can then be combined with other mission area analyses to provide an across the board evaluation of alternative force structures.

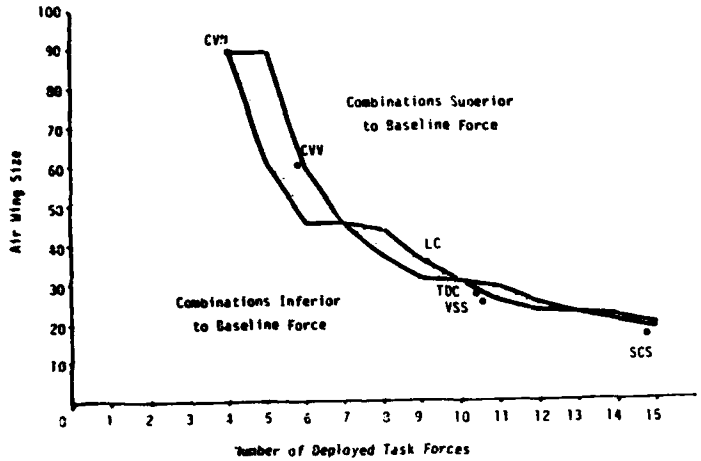

Figure 15. ALTERNATIVE EQUAL COST FORCE STRUCTURE COMPARISONS TO BASELINE, 3 AREAS

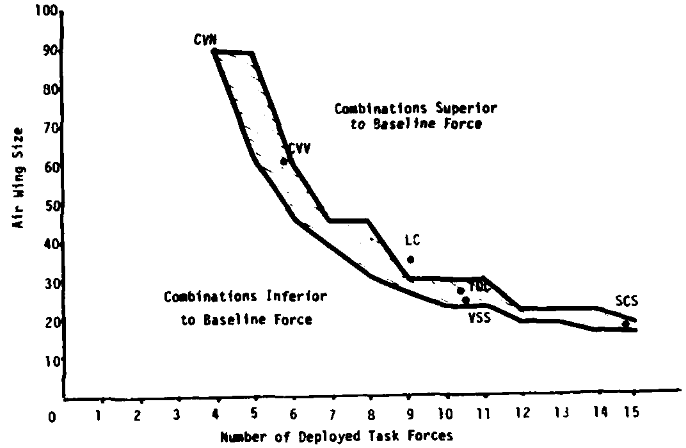

Figure 16. ALTERNATIVE EQUAL COST FORCE STRUCTURE COMPARISONS TO BASELINE, 4 AREAS

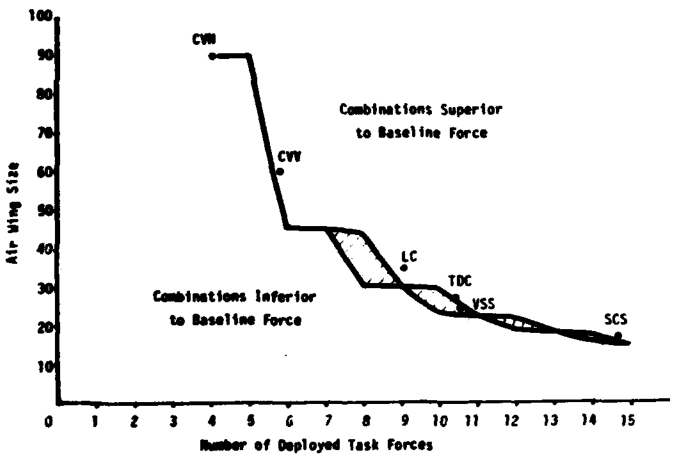

Figure 17. ALTERNATIVE EQUAL COST FORCE STRUCTURE COMPARISONS TO BASELINE, 5,6 AREAS

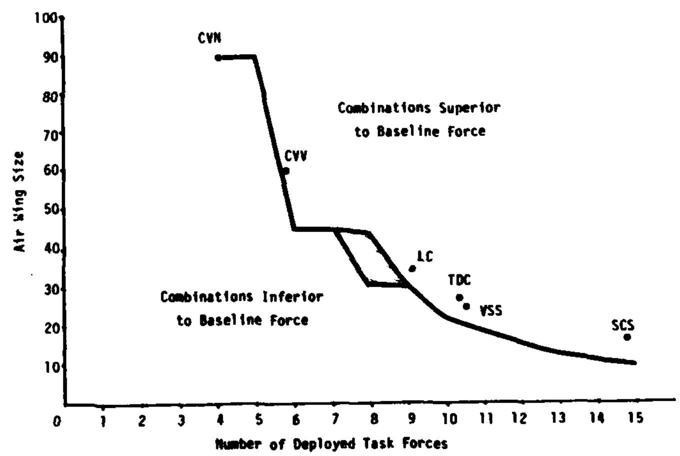

Figure 18. ALTERNATIVE EQUAL COST FORCE STRUCTURE COMPARISONS TO BASELINE, 7 AREAS

E. DISCUSSION AND DIRECTIONS FOR FURTHER WORK

The above results, while subject to certain limitations to be discussed further below, suggest the following observations.

1. As the number of world areas in which naval presence is desirable increases, the value of larger numbers of smaller carriers also increases, but only if the air wing size supported by those missions does not fall too quickly as the number of separate task forces increases.

2. The equal cost forces listed in Table 3 appear to follow fairly closely the ZSs of Figures 15, 16, and 17. Clearly, relatively small changes in the costing assumptions could move the CVV and LC points back into or below the ZSs. Thus, some readers might feel that the small carrier alternative forces show no consistent improvements over the baseline force until 7 presence areas are specified--over twice as many as the Navy must deal with today. This unexpected correspondence between the ZSs depicted in Figures 15-17 and the relationship between task force numbers and air wing sizes may shed some light on why small carriers appear to attract so little interest on the part of the Navy. This correspondence suggests that unless more efficient ways of dispersing sea-based aircraft to smaller carriers are found than have been proposed in the past, the advantages of small carriers for worldwide presence will remain uncertain. Certain caveats apply to this analysis, however.

First, this game theoretic measure of capability possesses a certain degree of artificiality in that it does not represent the simulating of actual confrontations between two powers. Rather, the measure assesses the ability of alternative force structures to place combat capability in any of a number of different areas in a zero-sum game context with respect to a given baseline force.

Second, while presence is desired in the different areas, it is not mandatory in this analysis. If mandatory presence in all ocean areas is specified, then some options become infeasible under a given budget limitation. For example, if mandatory presence is required for 7 areas, then under the cost constraint assumed implicitly in Figures 15-18 (based on 4 deployable CVNs), neither the CVN nor the CVV is a feasible option. Moreover, certain allocation strategies would be disallowed under this condition.

Third, the measure does not consider dynamic allocations of forces, that is, allocations that are functions of time as might be the case if certain task forces visited different areas in rotation.

Fourth, we are examining only presence, and not any of the other capabilities of carrier task forces such as crisis response and limited or general conflict. While this is not a limitation of the measure per se, one should bear in mind that this approach is valuable only as part of a broader analysis.

There are a number of areas for further work that have been identified in the course of preparing this paper. We have already noted that the results presented in this paper do not consider the capability of the aircraft that can be supported by the alternative carriers. Including such a measure of capability would be a worthwhile extension of this approach. It is likely that this would tend to reduce the relative desirability of the smaller carriers. Mixed forces with different carrier types are also handled, in theory, by the measure. The reduction procedures of Section C would have to be modified to handle mixed forces and it is likely that the degree to which the original game could be reduced would be less than in the pure force case.

There are also a number of theoretical issues that would be valuable to resolve. For example, while the procedure outlined in this paper is a valid mechanism for evaluating two force structures, we have not produced generalized results to allow one to extend that evaluation. Specifically, if force A is superior to force B in Ω oceans by this measure, and if force B is superior to force C in Ω oceans, we conjecture but have not been able to prove that force A is superior to force C in Ω oceans. While the question could be answered computationally, it would be convenient both from a practical point of view and as a justification of the measure to have a positive answer to this conjecture.

In spite of these limitations, we believe the measure is a useful one, helping to illuminate the capabilities of naval force structures to perform a noncombat but nonetheless important function.

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Appendix A

COSTING ASSUMPTIONS

COSTING ASSUMPTIONS

This Appendix outlines the costing assumptions used to generate the equal life cycle cost forces in Table 3.

Table A-1 lists the carrier options discussed in the text along with costs and some brief information concerning capabilities. Sources for this information are given in the footnotes to the Table. We assume that the 30-year life cycle costs of a carrier-type ship are approximately three times the procurement costs.

Two types of defensive escorts are employed in this analysis. For the CVN, in order to complement its nuclear propulsion capabilities, we provide the projected CGN-42 class ship--essentially a nuclear version of the CG-47 Aegis ship. The procurement cost of this type of ship is assumed to be $1.34 billion. For the other air-capable ships, we provide the proposed 8500 ton DDG-51 destroyer at a procurement cost of $0.71 billion. We further assume the 30-year life cycle cost of a surface combatant is approximately 2.5 times its procurement cost.

A number of factors enter into air wing costs including aircraft mixes, the number of procurements required to maintain one aircraft on deck for 30 years and the cost of individual aircraft types. From various sources, we estimate that the average 30-year life cycle cost of an aircraft in a CVN air wing is about $220 million. We use this basic cost to determine the cost of the alternative air wings. For those carriers that can handle only VSTOL aircraft, we increase the per-aircraft life cycle cost by 25 percent.

Table A-1. CARRIER OPTIONS

Ship Designation	Displacement (Long Tons)	Air Wing	Unit Procurement Cost (Millions of FY82 Dollars)	Remarks
Pure Carriers				
SCS (Sea Control Ship)	14,100[a]	17 Helicopters or VSTOL[b]	440[c]	Austere ship primarily for sea control.
VSS (VSTOL Support Ship)	26,000-29,000[d]	25 VSTOL[e]	980[f]	
LC (Light Carrier)	39,000[g]	35 VSTOL[h] or CTOL	1230[i]	This carrier could support contemporary CTOL aircraft such as the F/A-18, E-2C and S-3A, but not the F-14.[j] Possibly the A-6, modified for STOL operations, could also be accommodated by this carrier.
CVV	62,000[k]	60 VSTOL or CTOL[ℓ]	2252[m]	This ship could operate all current carrier aircraft.[n]
CVN	91,400[o]	90 VSTOL or CTOL[p]	3700[q]	
Air-Capable Ships With Additional Armament				
TDC (Through-Deck Cruiser)	45,000[r]	27 VSTOL[s]	1295[t]	Additional armament includes Harpoon for ASuW, ASROC for ASW, standard surface-to-air missiles (but AAW weapon system is unspecified), a twin 203 mm gun mount and six 76 mm guns.[u]

(Footnotes on next page)

Footnotes to Table A-1

[a]Department of Defense Appropriations for Fiscal Year 1973. Hearings Before a Subcommittee of the Committee on Appropriations, United States Senate, Ninety-Second Congress, Second Session, Part 3, p.939.

[b]Ibid.

[c]Ibid. The cost of the SCS given in this testimony is $103.1 million compared to $797 million for CVN-70 (in then-year dollars). Scaling both ships to FY 82 dollars yields about $440 million for the SCS.

[d]Assessment of Sea Based Air Platforms Project Report, Office of the Secretary of the Navy, Department of the Navy, Washington, D.C., February 1978. The higher figure is for a design with greater passive defenses.

[e]Ibid. We use the midpoint of the 24 to 26 aircraft range indicated.

[f]Department of Defense Appropriations for Fiscal Year 1976. Hearings Before a Subcommittee of the Committee on Appropriations, United States Senate, Ninety-Fourth Congress, First Session, Part 3--Department of the Navy. Page 330 of this testimony presents VSS designs with slightly greater displacement than the one we present. Based on the costs presented there, we estimate the ten-year follow-ship procurement cost of the VSS to be about $400 million in FY 76 dollars. This compares with $180.2 given for the SCS in the same testimony. Using the $440 million in FY 82 dollars already derived for the SCS, adjusting the VSS cost to FY 82 dollars yields about $480 million.

[g]Department of Defense Authorization for Appropriations for Fiscal Year 1981. Hearings before the Committee on Armed Services, United States Senate, Ninety-Sixth Congress, Second Session, Part 2, p. 1102.

[h]Ibid.

[i]Ibid. Cost figures are not given, but the Navy's goal "...is one-third the price of a CVN in the year of authorization." We use $3.7 billion for CVN unit cost.

[j]Ibid.

[k]Department of Defense Authorization for Appropriations for Fiscal Year 1982. Hearings Before the Committee on Armed Services, United States Senate. Ninety-Seventh Congress, First Session, Part 4--Sea Power and Force Projections, p. 2038.

[ℓ]Ibid.

[m]Ibid.

Footnotes to Table A-1 (continued)

[n]Full Committee Consideration of the CVV Program. Committee on Armed Services, House of Representatives, Ninety-Fifth Congress, First Session, May 24, 1977, p.9.

[o]Moore, John E., ed., "Jane's Fighting Ships," 1977-1978, p.570.

[p]Ibid.

[q]Mitchell, Douglas, D., "Shipbuilding Costs for General Purpose Forces in a 600-Ship Navy," Congressional Research Seminar Report 82-23F, February 16, 1982, p.CRS-20.

[r]Cairl, Michael A., "Through Deck Cruiser: The New Capital Ship," U.S. Naval Institute Proceedings, December 1978, p.39.

[s]Ibid. A range of 25-30 aircraft is specified.

[t]Ibid. The cost is specified as "35% of the cost of [a] nuclear-powered carrier."

[u]Ibid.

Table A-2 summarizes life cycle costs of alternative carrier task forces. The footnote to the table summarizes the procedure used to determine underway replenishment costs. These costs are then used to determine the number of task forces equal in 30-year costs to 4 CVN task forces.

Table A-2. LIFE CYCLE COSTS OF ALTERNATIVE TASK FORCES

	Ship Cost[a]	Air Wing Cost[a]	Defensive Escort Cost[a]	Underway Replenishment Cost[a]	Total Cost
	Millions of FY 1982 Dollars				
CVN	11100	19800	6700	5526	43126
SCS	1320	4675	3552	2105	11652
VSS	2940	6875	3552	3052	16419
LC	3690	7700	3552	4059	19001
CVV	6750	13200	3552	6378	29880
TDC	3900	7425	1776	3482	16583

[a]From various sources, we estimate that $14.42 billion in underway replenishment assets (life cycle costs) can be ascribed to the Navy's notional surface combatant force level battle group built around two carriers and containing 12 conventionally powered ships and 180 aircraft. We assume that propulsion and aviation fuel drive replenishment costs. Of petroleum used by the Navy for ships and aircraft, ships use about 52 percent of the total, aircraft about 48 percent (Collins, Frank C., "Energy: Essential Element of National Security," U.S. Naval Institute Proceedings, December 1980). The Navy operates ships totalling about 5,400,000 tons displacement ("Jane's Fighting Ships 1977-78") and about 2700 Navy and Marine Corps aircraft ("The Military Balance," 1981-82) so that if underway replenishment requirements for ships scale proportionally to displacement, each 1000 tons of displacement accounts for about .010 percent of the fuel required by ships and aircraft, while each aircraft accounts for about .018 percent. In other words, each aircraft equals about 1.8 kiloton of displacement in terms of underway replenishment requirements. In the battle group cited, therefore, 180 aircraft account for 324 kilotons of displacement while the conventionally propelled escorts account for only about 99 kilotons of displacement. Therefore each displacement kiloton costs about 34.1 million in 30-year replenishment costs, each aircraft about 61.4 million. These calculations are, of course, very crude but we believe them adequate for our purposes.

More precise costing would be desirable in a more thorough analysis. We believe, however, that the figures cited in this Appendix are reasonably accurate and for the purposes of this paper reflect adequately the relationship between numbers of aircraft and numbers of deployable task forces.

Appendix B

TABULATED RESULTS

TABULATED RESULTS

This Appendix tabulates the results shown in Figures 11-14 of the major text. The information is presented in this form for those who would like to examine more exact values than are easily obtainable from the Figures.

Table B-1. LIMITS ON NUMBERS OF AIRCRAFT PER CARRIER THAT YIELD ZERO GAME VALUE FOR CONDITIONS INDICATED

Number of Areas	Number of Task Forces: 4	5	6	7	8	9	10	11	12	13	14	15
3	90/90	61/89	46/60	45+/46-	37/44	31/36	30+/31-	26/29	23/25	22+/23-	21/22	19/20
4	90/90	61/89	46/60	45+/46-	31/45	30+/31-	23/30	23/30	19/22	19/22	16/22	16/18
5	90/90	90/90	45+/46-	45+/46-	31/44	30+/31-	23/30	22+/23-	19/22	18+/19-	16/18	15+/16-
6	90/90	90/90	45+/46-	45+/46-	31/44	30+/31-	23/30	22+/23-	19/22	18+/19-	16/18	15+/16-
7	90/90	90/90	45+/46-	45+/46-	31/44	30+/31-	22+/23-	18+/19-	15+/16-	12+/13-	11+/12-	10+/11-

Explanation of Entries:

a/b signifies that for all integer values of aircraft I such that $a \le I \le b$, the number of task forces indicated by the column heading, each supporting I aircraft, yields a zero game value when assessed against four 90-aircraft task forces in a game involving the number of areas specified by the row heading. Values outside that range yield positive or negative values. Entries of the form a+/(a+1)- signify that no integer value yields a zero game value, but that for integers $I \ge a+1$, a positive game value is obtained, while for integers $I \le a$, a negative game value is obtained.